

Exercise 18

Find an expression for the function whose graph consists of the line segment from the point $(-2, 2)$, to the point $(-1, 0)$ together with the top half of the circle with center the origin and radius 1.

Solution

Start by finding the line $y = mx + b$ that goes through $(-2, 2)$ and $(-1, 0)$.

$$2 = m(-2) + b$$

$$0 = m(-1) + b$$

Solve this system of equations for m and b : $m = -2$ and $b = -2$. So the line is

$$y = -2x - 2.$$

The equation for a circle centered at the origin in the xy -plane with radius 1 is

$$x^2 + y^2 = 1.$$

Solve for y .

$$y^2 = 1 - x^2$$

Take the square root of both sides.

$$|y| = \sqrt{1 - x^2}$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$y(x) = \pm\sqrt{1 - x^2}$$

Since we want the top half of the circle, we choose the plus sign.

$$y = \sqrt{1 - x^2}$$

Therefore, the expression for the function in question is

$$f(x) = \begin{cases} -2x - 2 & \text{if } -2 \leq x \leq -1 \\ \sqrt{1 - x^2} & \text{if } -1 \leq x \leq 1 \end{cases}.$$

